Writing Solutions

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We can categorize problems into a couple different types based on what is required. There are those problems which tell you a statement that you have to prove, and those where you have to make your own statement and then justify it.

In either case, we see that it is required to be able to write good solutions. For this reason, it is important that you first solve the problem and then proceed to write out the solution.[1] Here are some tips to remember:

- Consider an outline or overview for long/difficult proofs (not required for proofs in contests which are always short).
- Consider splitting your proof into smaller lemmas which are individually proved.
- Use visual demarcation (such as indents, spacing, and underlining) to denote separate parts of the proof and maintain a hierarchy when the casework has subcases.
- Explicitly denote the end of a proof with "QED" or a small box.
- Explicitly state your statement/claim at the beginning of the solution and optionally box it at the end.

For example, consider a question which asks us to find the smallest triangular number greater than 100. If you are unfamiliar with triangular numbers, they can be defined quite easily.

Definition 1. *The n^{th}* ***triangle number****, T_n, is the number of dots required to form an equilateral triangle out of dots with n dots on each side. Here are the first few triangular numbers which can be explicitly calculated with the formula $T_n = \frac{n(n+1)}{2}$.*

n	0	1	2	3	4	5
T_n	0	1	3	6	10	15

Here's an example of how you might format the solution to this problem:

Solution. The smallest triangle number greater than 100 is 105.

Proof. A triangular number is of the form $\frac{n(n+1)}{2}$ for some positive integer n, and we can see that this expression is strictly increasing with respect to n for $n \geq 0$. Thus, we see that since $\frac{13(13+1)}{2} = 91 \leq 100 < 105 = \frac{14(14+1)}{2}$, the 14^{th} triangle number, $\boxed{105}$, is the smallest triangle number greater than 100. □

1. But if you are writing a contest where you do not have enough time to solve the problem, it is better to write what you have than nothing at all.